OPTIMAL CONTROL OF ONE-DIMENSIONAL IMPACT ONTO AN INHOMOGENEOUS POROUS MATERIAL

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The propagation of a strong shock wave in a porous material takes place when the latter is used as a protective shield against shock or explosion, and also in powder metallurgy during the dynamic impact or explosive forming of porous metals and metal powders [1]. It is often necessary to optimize particular structures or processes in such cases [2]. In the present article we formulate a number of optimal control problems on the basis of a theoretical analysis of one-dimensional impact onto an inhomogeneous porous material and give a solution of the kinetic-energy optimization problem for an inhomogeneous porous barrier.

STRONG SHOCK WAVE IN A POROUS MATERIAL

The compaction of a porous material is customarily described by equations of state that have a complicated form and a restricted domain of application [3, 4]. The situation is simplified in the case of a strong shock wave, in which the pores collapse irreversibly during compaction. To describe the flow after the first shock wave we use the equation of state for a nonporous material. We assume that the subsequent variation of the specific volume is small in comparison with the specific volume of the nonporous material. We introduce the following notation: the pressure p_1 in a shock wave that compacts the porous material to the density of the nonporous material under standard conditions; a characteristic pressure p_2 such that the specific volume of the continuous material undergoes, say, a twofold variation. The relation $p_1 \ll p_2$ holds for many materials. For example, in the case of iron with a porosity (ratio of the density of the porous material to the density of the nonporous material) in the interval 0.6-1, we have $p_1 \sim 1$ GPa [5] and $p_2 \sim 10^2$-10^3 GPa. Consequently, the pressure range in which the above-stated assumptions are valid is fairly broad (see also [6]).

We consider a strong shock wave in a porous material having an initial volume $v_0 = v_0/\alpha$, where v_0 is the specific volume of the nonporous material under standard conditions and α is the porosity. The initial values of the pressure and internal energy are taken equal to zero, and the surface energy of the pores (or powder grains) is neglected. When the nonporous material obeys the relation $D = c + su$ (D is the velocity of the shock front, u is the particle velocity behind the front, and c, s are constants), we obtain a shock-adiabatic equation that well describes the compression of porous materials in a strong shock wave:

$$p_H = \frac{c^2 (v_0 - v_H)}{[v_0 - s(v_0 - v_H)]^2} \frac{1 - \frac{\gamma_0}{2v_0}(v_0 - v_H)}{1 - \frac{\gamma_0}{2v_0}(v_1 - v_H)}, \tag{1}$$

where v_H and p_H are the specific volume and pressure on the shock adiabat and γ_0 is the Grüneisen constant under standard conditions.

In the pressure range of interest

$$|v_0 - v| \ll v_0. \tag{2}$$

We shall use Lagrangian coordinates (m, t) below, where m is the total mass of one square meter of the wall layer between the boundary of the barrier and a given cross section, and t is the time. Let m_S be the Lagrangian coordinate of the shock front. An expression for the velocity of the shock front in the porous material follows from Eqs. (1) and (2) and the relations at the shock front:

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$$\dot{m}_S = a\left(\frac{p_H}{p_H+\delta}\right)^{1/2},\quad a=\frac{c}{v_0},\quad \delta=\frac{a^2 v_0(1-\alpha)}{\alpha}. \tag{3}$$

After the compaction of the porous material by the shock wave, its subsequent behavior is adequately described under condition (2) by the isentrope passing through the point (p_H, v_H). From the isentropicity condition $dE = -pdv$, Eq. (1), and the equation of state used in [5] for a nonporous material we obtain the isentropic equation for the shock-compressed porous material in the approximation (2):

$$p = a^2(v_0 - v) + \frac{\gamma_0}{2}\frac{1-\alpha}{\alpha}p_H. \tag{4}$$

Equations (3) and (4) are valid for a material of arbitrary porosity.

ONE-DIMENSIONAL IMPACT ONTO AN INHOMOGENEOUS POROUS MATERIAL

For the postshock region we write the equations

$$\partial v/\partial t = \partial u/\partial m,\ \partial u/\partial t = -\partial p/\partial m. \tag{5}$$

Eliminating u and v from Eqs. (4) and (5), we obtain equations describing the pressure behind the shock wave and the motion of the shock front itself:

$$\partial^2 p/\partial t^2 = a^2\partial^2 p/\partial m^2,\ t \geqslant 0,\ m \in [0,\ m_S(t)]; \tag{6}$$

$$p(0, t) = \omega(t); \tag{7}$$

$$\dot{m}_S = \chi(p(m_S, t), m_S),\ m_S(0) = 0; \tag{8}$$

$$\chi(p, m) = a\left(\frac{p}{p+\delta(m)}\right)^{1/2},\quad \delta(m) = \frac{a^2 v_0(1-\alpha(m))}{\alpha(m)}. \tag{9}$$

Here $m_S(t)$ is the Lagrangian coordinate of the front, $\omega(t)$ is the specified pressure on the left boundary of the barrier, and $\alpha(m)$ is the specified dependence of the material porosity on m. We assume that the inhomogeneity of the material is contained entirely in the function $\alpha(m)$. We note that the pressure field behind the shock wave is described by the wave equation (6) with a = const, despite the nonuniform heating of the shock-compressed material.

Inasmuch as the right boundary of the flow region is unknown, Eq. (8) must be augmented with an additional condition at this boundary. For its derivation we calculate the velocity increment du at the right boundary $m_S(t)$ during the time dt in two ways, expressing the increment each time in terms of the pressure and its derivatives. This can be done, on the one hand, by means of Eqs. (4) and (5) and, on the other, by invoking the Hugoniot relations. Equating the resulting equations and making necessary transformations, we find the condition at the unknown boundary

$$\partial p(m_S, t)/\partial t + \psi\partial p(m_S, t)/\partial m = \varphi\alpha'(m_S), \tag{10}$$

where

$$\psi = \psi(p, m) = \frac{4p+3\delta(m)}{4p+\delta(m)}\chi(p, m); \tag{11}$$

$$\varphi = \varphi(p, m) = \frac{a^2 v_0}{\alpha^2(m)}\frac{p}{4p+\delta(m)}\chi(p, m). \tag{12}$$

We write the solution of Eq. (6) in the form

$$p(m, t) = f(t - m/a) + g(t + m/a), \tag{13}$$

where f and g are arbitrary functions. It follows from Eqs. (8) and (9) that $\dot{m}_S \leq a$, so that for $m \in [0, m_S(t)]$, $t \geq 0$ (i.e., in the postshock flow region) the arguments of the functions f and g are nonnegative. It is clear from (7) that $g(t) = \omega(t) - f(t)$, and so

$$p(m, t) = f(t - m/a) - f(t + m/a) + \omega(t + m/a). \tag{14}$$

Substituting the latter relation in condition (10), we reduce the basic problem to the problem for a system of differential equations with one independent variable and a neutral-type deviating argument [7]. A suitable candidate for the independent variable is the quantity $\xi = t + m_S(t)/a$, and we introduce the notation $\eta(\xi) = 2m_S(t)/a$. We arrive at the problem for a system of two equations with two unknown functions $\eta(\xi)$ and $f(\xi)$ (the prime denotes differentiation with respect to the independent variable):

$$\eta'(\xi) = 2 - \frac{2a}{a+\chi}; \tag{15}$$

$$f'(\xi) = \omega'(\xi) + \frac{a-\psi}{a+\psi} f'(\xi - \eta(\xi)) - \frac{a\varphi\alpha'}{a+\psi}. \tag{16}$$

Here $\xi \geq 0$; the initial set consists of the one point $\xi = 0$; the conditions on the initial set are $\eta(0) = 0$, $f(0) = 0$ [the latter condition is adopted for convenience, but in reality the value of $f(0)$ can be arbitrary]. On the right-hand sides of Eqs. (15) and (16), χ, ψ, and φ are two-variable functions specified by Eqs. (9), (11), and (12); the values of the arguments of these functions are equal to $f(\xi - \eta(\xi)) - f(\xi) + \omega(\xi)$ and $\alpha\,\eta(\xi)/2$, respectively. The value of the argument of the function α' in (16) is equal to $\alpha\,\eta(\xi)/2$.

The solution of problem (15), (16) exists and is unique [8]. This problem can be solved numerically by varying ξ with a certain step, but all the computed values of the function $f(\xi)$ and its derivative must be stored; the values of f and f' required in each step for the delayed values of the argument can be computed by interpolation. Once the function $f(\xi)$ has been found, the pressure field is determined according to (14). The method described here has the drawback that the functions $\omega(t)$ and $\alpha(m)$ must be smooth.

Another approach to the solution of the basic problem (6)-(12) is possible, representing a variant of the method of the characteristics. In this approach, the functions $\omega(t)$ and $\alpha(m)$ are assumed to be piecewise-constant (the number and size of the jumps are arbitrary). The functions f and g in Eq. (13) are also piecewise-constant. The f-jumps (i.e., the jumps of the function f) can be of two kinds, depending on their origin, namely they can be generated by ω-jumps or they can be induced by the reflection of g-jumps from the left boundary. Similarly, two kinds of g-jumps can occur: those generated by α-jumps at the arrival time of the shock wave and those generated by f-jumps at the instant when they overtake the shock wave. The expressions for the sizes of the f- and g-jumps follow from the boundary conditions.

Consequently, in place of the basic problem we have the problem of describing the motion of a system of jumps. This problem can be solved numerically by varying the time with a certain step and computing the new jump parameters (with allowance for the generation of new jumps) and the coordinate of the shock front each time. The pressure field is determined according to (13) in each step.

From the integral form of the momentum conservation law we obtain an equation for the velocity field:

$$u(m,t) = u(m_S,t) + \frac{1}{a}\left[f\left(t - \frac{m}{a}\right) - f\left(t - \frac{m_S}{a}\right)\right] - \frac{1}{a}\left[g\left(t + \frac{m}{a}\right) - g\left(t + \frac{m_S}{a}\right)\right],$$

where $u(m_S,t) = \frac{1}{a}[p_H(p_H+\delta)]^{1/2}$; the latter expression follows from (8) and (9), and the Hugoniot relations.

From Eq. (4) and the relations at the shock front we deduce an expression for the specific internal energy at an arbitrary point of the flow:

$$E(m,t) = \frac{1}{2a^2}[p(m,\tau)\,\delta(m) + p^2(m,t)], \tag{17}$$

where τ is the solution of the equation

$$m_S(\tau) = m. \tag{18}$$

In regard to the evaluation of the shock-damping efficiency of a porous wall it is instructive to consider the total energy converted irreversibly into heat or, more precisely, the total internal energy of the wall during its unloading (p = 0 for all m):

$$W = \frac{1}{2a^2}\int_0^M p_H(m)\,\delta(m)\,dm, \tag{19}$$

where M is the Lagrangian coordinate of the end of the wall, $p_H(m) = p(m, \tau)$, and $\tau(m)$ is the solution of Eq. (18).

We give the results of a calculation of the action of a loading pulse on a three-layer wall, for which

$$\alpha(m) = \begin{cases} 1 & \text{for } m \in [0, 100), \\ 0.5 & \text{for } m \in [100, 200], \\ 1 & \text{for } m \in (200, 300]. \end{cases} \tag{20}$$

The wall material is iron (with a porous middle layer), $a = 2.91 \cdot 10^7$, $v_0 = 1.27 \cdot 10^{-4}$, M = 300 (corresponding to a wall thickness of 5.08 cm), and all dimensional quantities, unless otherwise indicated, are given in SI units. The pressure pulse is rectangular with an amplitude $p_0 = 10^{10}$ and a duration $\tau = 10^{-5}$.

The total energy transmitted to the wall by the pressure pulse is

$$K = \int_0^\tau p(0, t)\,u(0, t)\,dt. \tag{21}$$

As a measure of the damping efficiency of the wall we adopt the kinetic energy remaining at the wall after its shock compaction (the lower the energy, the greater will be the shielding efficiency): I = K − W [see Eq. (19)]. We recall that the total momentum of the wall from the time of termination of the loading pulse up to the start of interaction of the wall with other objects remains constant.

For the three-layer wall (20), $K = 4.99 \cdot 10^7$, $W = 2.30 \cdot 10^7$, and $I = 2.69 \cdot 10^7$; for comparison we point out that in the case of a homogeneous wall of a nonporous material having the same mass (thickness 3.81 cm), $I = K = 3.44 \cdot 10^7$, and W = 0. For a homogeneous porous wall having the same mass and thickness as the inhomogeneous wall (20), corresponding to $\alpha = 0.75$, we obtain $K = 7.35 \cdot 10^7$, $W = 5.61 \cdot 10^7$, and $I = 1.74 \cdot 10^7$. It is evident from these results that the shock-damping efficiency of porous walls can be appreciable.

PROBLEMS IN THE OPTIMAL CONTROL OF IMPACT

The problem of impact onto an inhomogeneous porous wall has been formulated above in three different forms: A) a boundary-value problem for the wave equation with an unknown boundary, Eqs. (6)-(12); B) the same for a system of differential equations with a deviating argument, Eqs. (15), (16); C) the motion of a system of discontinuities (jumps). All three forms are equivalent from the physical point of view, and so for definiteness we proceed from problem A. A number of optimal control problems can be formulated on its basis, corresponding to different situations. We now discuss some of them. (Everywhere below we take our control functions from a certain sensibly chosen class of functions, e.g., from the set of piecewise-differentiable functions specified on an interval; we shall not make specific reference to the class of allowed control functions below.)

1. Let us consider the damping of an impulse load on a porous wall. We state the problem of finding the optimum wall structure, i.e., the structure for which the kinetic energy remaining at the wall after shock transmission is a minimum for a given pressure pulse. In this situation the function $\omega(t)$ in problem A is specified, and $\alpha(m)$ is an unknown control function. We have

$$I = K - W \to \min, \tag{22}$$

where K and W are calculated according to Eqs. (21) and (19). We formulate the constraints on the values of the porosity α and on the total size of the wall:

$$\alpha(m) \in [\alpha_{\min}, 1]; \tag{23}$$

$$l = v_0 \int_0^M \frac{dm}{\alpha(m)} \leqslant l_{\max}. \tag{24}$$

We thus have the optimal control problem (6)-(12), (22)-(24).

2. We can also formulate the inverse of problem 1: to determine the optimum pressure pulse in the penetration of a porous wall with a given structure. In this situation, the kinetic energy of the wall is maximized, $\alpha(m)$ is given, and $\omega(t)$ is the control function. The constraints on the admissible control functions can differ here, depending on the specific physical situation.

3. Let us consider the following modification of problem 1: to determine the damping of impact loading by a porous wall when the driver is an undeformable solid plate (in which the sound velocity is infinite) with a given mass M_p and initial velocity u_{p0}. In this case, the condition (7) on the left boundary in the basic problem A is replaced by the condition

$$p(0, t) + M_p \partial u(0, t)/\partial t = 0, \; u(0, 0) = u_{p0},$$

which follows from the equation of motion of the solid driver plate. The control function is the structure of the wall, $\alpha(m)$. The total kinetic energy of the wall and the driver plate is minimized:

$$I = \frac{1}{2} M_p u_{p0}^2 - W \to \min. \tag{25}$$

(This optimality criterion is similar to the one used in [2].) The constraints on α are the same as in problem 1.

4. We now consider the problem of optimizing the damping of impact by a porous wall when the driver is also a porous plate (the structure of the driver plate is given). In this case, we replace the conditions (7) and $u(m, 0) = 0$ in the basic problem A by the initial and boundary conditions

$$p(0, t) = 0, \; u(m, 0) = \begin{cases} u_{p0}, & m \in [0, M_p), \\ 0, & m \in (M_p, M]. \end{cases}$$

The driver plate corresponds to the Lagrangian coordinate interval $[0, M_p]$, and the target plate to the interval $[M_p, M]$. The flow region has two unknown boundaries $m_{Sp}(t)$ and $m_S(t)$, which correspond to two shock waves (in the driver and in the target). Here $\alpha(m)$, $m \in [0, M_p]$, is the given structure of the driver plate and $\alpha(m)$, $m \in [M_p, M]$, is the control function (structure of the target plate). The total kinetic energy of the driver and the wall is minimized according to Eq. (25). The constraints on the control function are the same here as in problem 1.

5. We can formulate the inverse of problem 4: to determine the optimum structure of the driver in penetrating a porous wall with a given structure when the driver is also a porous plate.

6. Let us consider the problem of optimizing the shock heating of a porous material, viz.: to determine for a given loading pulse (7) the wall structure $\alpha(m)$ for which the average temperature of a certain given part of the wall $[m_1, m_2]$ is a maximum after shock compaction:

$$W_1 = \int_{m_1}^{m_2} p_H(m) \delta(m) \, dm \to \max,$$

where $p_H(m)$ is the same as in Eq. (19).

7. A problem that arises in powder metallurgy in connection with the dynamic forming of porous metals and metal powders by a strong impact or explosion is the optimization of the structure of the initial porous blank from the point of view of eliminating residual stresses in the end product. For example, if we know the distribution of the internal energy (temperature) $E_0(m)$ in the compacted metal for minimization of the internal stresses after cooling of the product, we can formulate the problem of determining the structure of the blank

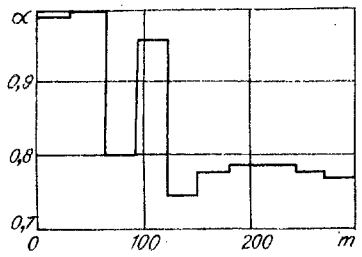

Fig. 1

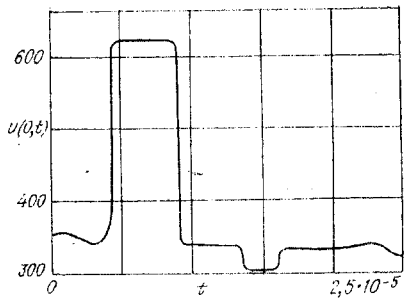

Fig. 2

so as to minimize the deviation of the internal energy distribution of the material after compaction from the distribution $E_0(m)$:

$$I = \int_0^M \left[\frac{1}{2a^2} p_H(m)\,\delta(m) - E_0(m)\right]^2 dm \to \min.$$

Here $\omega(t)$ is a given function, and $\alpha(m)$ is the control function (with the same constraints as in problem 1). The condition $u(M, t) = 0$ can be used at the right boundary of the wall.

We now consider the numerical solution of problem 1 (a similar approach can be applied to problems 2-7). Going over to the discrete analog of problem 1, we consider a porous wall consisting of n homogeneous layers of equal mass with porosities $\alpha_1, \ldots, \alpha_n$. Here we use problem C in place of the basic problem A. This reduces the optimal control problem to the problem of minimizing the function of n variables $I(\alpha_1, \ldots, \alpha_n)$. The specific computation of I is carried out by solving problem C numerically.

To solve the resulting mathematical programming problem we use the method of Hooke and Jeeves [10] modified with regard for the constraints implied by (23) and (24):

$$\alpha_i \in [\alpha_{\min}, 1],\ i = 1, \ldots, n; \tag{26}$$

$$f(\alpha) = \sum_{i=1}^{n} \frac{1}{\alpha_i} \leqslant A = \frac{l_{\max}}{v_0 \Delta m}, \tag{27}$$

where Δm is the specific mass of each layer. The constraints (26) and (27) can be taken into account either by the method of penalty functions [9, 11] or by the technique described below.

In the Hooke–Jeeves algorithm the coordinates $\alpha_1, \ldots, \alpha_n$ are incremented successively by the amount $\pm\varepsilon$ ($\varepsilon > 0$). If the increment of the k-th coordinate does not decrease the objective function, it is rejected (i.e., the previous value of α_k is restored). Coordinate increments that violate at least one of the constraints (26), (27) are also rejected (the same thing would happen with the introduction of penalty functions, i.e., terms in I that grow rapidly when the constraints are violated.

The modification of the Hooke–Jeeves algorithm entails motion of the operating point parallel to the bounding hypersurface in the event of proximity to it, if the above-described steps do not result in a decrease of I. If the operating point is situated near the bounding hypersurface $f(\alpha) = A$, there always exists at least one coordinate α_k that can be varied in both directions, i.e., the operating point is situated far from the bounder hyperplanes $\alpha_k = \alpha_{\min}$ and $\alpha_k = 1$. In this case, if the successive increments of all the variables $\alpha_1, \ldots, \alpha_n$ are nonproductive (i.e., the value of I does not decrease), then a set of steps of the following form is made successively for all $l \neq k$:

$$\Delta\alpha_k = \pm\varepsilon\left(1 + \frac{\alpha_l^4}{\alpha_k^4}\right)^{-\frac{1}{2}},\quad \Delta\alpha_l = \left(\frac{1}{\alpha_k} + \frac{1}{\alpha_l} - \frac{1}{\alpha_k + \Delta\alpha_k}\right)^{-1} - \alpha_l. \tag{28}$$

The step (28) represents a shift of the Euclidean length ε in the two-dimensional plane (α_k, α_l), such that the point remains on the surface $f(\alpha) = \text{const}$. If successive steps of the

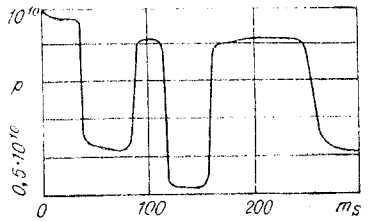

Fig. 3

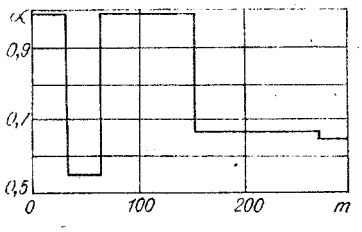

Fig. 4

form (28) also fail to yield a result, the size of the step ε is diminished, and convergence is tested (the operating point is assumed to be the minimum point when ε is smaller than the prescribed error).

Following are the results of computations of the optimum structure of a damping porous wall for a rectangular pressure pulse with amplitude $p_0 = 10^{10}$ and duration $\tau = 10^{-5}$. The material of the wall is iron, and the specific mass of the wall M = 300. The admissible values of the porosity are $\alpha = 0.25\text{-}1$. The wall thickness has the restriction $\ell_{max} = 5.08$ cm (corresponding to the thickness of a homogeneous wall with $\alpha = 0.75$). The α thickness is taken equal to 0.01.

The computations for a three-layer structure (n = 3) gave the following optimum parameters: $\alpha_1 = 0.98$, $\alpha_2 = 0.75$, $\alpha_3 = 0.78$, $I = 1.67\cdot10^7$, $\ell = 4.62$ cm.

We next carried out a more refined computation (n = 10). Since the Hooke–Jeeves algorithm implements a search for a local minimum, the minimum point can depend on the choice of initial approximation. It is evident from the computations that the optimum wall structure and the value of the objective functional are practically the same for a set of distinctly different initial approximations. Figure 1 shows the optimum structure of the porous wall for the initial approximation $\alpha(m) = 0.9 = \text{const}$ ($I = 2.98\cdot10^7$). The values of I and ℓ in this case are practically the same as for the optimum three-layer structure. Figures 2 and 3 show the computed dependences of the velocity of the loaded surface on the time and of the postshock pressure on the coordinate of the shock front for a porous wall with the optimum structure.

The computations were repeated for various amplitudes and durations of the pressure pulse: $p_0 = (0.5\text{-}2)\cdot10^{10}$, $\tau = (0.75\text{-}1.5)\cdot10^{-5}$. It was found that for small values of the product $p_0\tau$ (the total impulse transmitted to the wall) the optimum wall is practically a homogeneous wall with $\alpha \approx 0.75$. The optimum wall structure shown in Fig. 1 is typical for intermediate values of the product $p_0\tau$. Finally, with a further increase in $p_0\tau$ the optimum structure is typified by "broadening" of the maximum of $\alpha(m)$ in the middle part of the wall. Figure 4 shows the optimum structure of a porous wall for $p = 2\cdot10^{10}$, $\tau = 1.25\cdot10^{-5}$, and the initial approximation $\alpha(m) = 0.76 = \text{const}$ ($I = 1.62\cdot10^8$). The optimum wall corresponds to a thickness $\ell = 5.07$ cm and a value of the objective functional $I = 1.21\cdot10^8$. With an increase in the total impulse $p_0\tau$ the gain in the value of the objective functional for the optimum structure (in comparison with a homogeneous porous wall for $\alpha = 0.75$) improves considerably.

Optimal control problems analogous to those discussed here can be formulated on the basis of a theory that admits partial compaction of a porous material in a shock wave [12, 13].

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SHOCK WAVE STRUCTURE IN A MIXTURE OF GAS AND MELTING PARTICLES

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The process of fusion of solid particles dispersed within a gas flow occurs during various gasdynamic reactions in technical equipment. In particular, for noncalculable regimes in Laval nozzles, using a gas with solid combustion products of some fuel as a working medium, the structure of the shock wave which develops is complicated as compared to that of a shock wave propagating in a mixture of gas and particles without consideration of the phase transition, because aside from the relaxation process of equalization of the temperatures of the phases, particle transition from the solid to the liquid state occurs with a finite relaxation time. The driving force for this transition is the difference of the liquid concentration from its equilibrium value.

The structure of a shock wave in a mixture of gas with melting particles was studied in [1, 2] within the framework of single-velocity, single-temperature mechanics of heterogeneous media with consideration of the nonequilibrium fusion process. The case in which the process of heat exchange between the phases occurs at a finite rate requires consideration based on a model which considers the difference between the temperatures of the phases. At the same time, considering the particles to be sufficiently small, and assuming that they are instantaneously carried off by the gas flow, we may conclude that the simplified model proposed in [3] will be adequate for the study of shock wave propagation in a mixture of gas with metal particles, with consideration of fusion. We will assume that the heat of phase transition L is independent of the fusion temperature, determined by the pressure of the mixture.

1. Formulation of the Problem of Determination of Shock Wave Structure in the Mixture. Study of the Hugoniot Adiabat. We will consider the process of shock wave propagation in a mixture of gas and solid particles. The equations describing this phenomenon in a reference frame traveling with the shock wave have the form

$$\rho u = c_1,\ p + c_1 u = c_2,\ e + pv + u^2/2 = c_3, \qquad (1.1)$$
$$p = \frac{\bar{\alpha} RT}{w},\ e = \bar{c}_{V1} T + c_* T_2 + L\xi,$$
$$u\dot{\xi} = \varkappa,\ u\dot{T}_2 = q,\ \rho_{22} = \rho_{33} = r$$

(using the notation of [3]). We will use the source function

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